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A GENERALIZATION OF OROWAN'S APPROACH FOR PLANE STRAIN ROLLING TO AXISYMMETRIC EXTRUSION AND DRAWING

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ABSTRACT

Approximate theoretical methods can be successfully used for preliminary design of metal forming processes. In utilizing such methods, it is especially important to take into account the behaviour of the real functions that must exist where strong local effects are of importance. A typical example in this respect is the vicinity of frictional interfaces. It is known that the real velocity field next to a surface of maximum friction stress must generally involve non-differentiable functions when a class of rigid plastic models is adopted. The main idea of the method proposed by Orowan for plane strain rolling makes it very suitable for accounting for such solution behaviour. In the present paper a generalization of Orowan's method to axisymmetric extrusion and drawing is developed. The asymptotic behaviour of the real velocity field in the vicinity of the friction surface is taken into account in the approximate method. Therefore, it is possible to evaluate the strain rate intensity factor and its dependence of process parameters. By assumption, this factor controls the evolution of material properties in a narrow layer in the vicinity of surfaces with high friction. Therefore, the method developed has a potential for its application to metal forming design driven by material properties in a thin sub-surface layer.

Key words: *Orowan's method, velocity field, axisymmetric extrusion and drawing*

1. INTRODUCTION

There are a number of methods used for finding approximate solutions for metal forming processes [1-3]. Each of these methods is applicable to a wide class of processes. In contrast to these methods, the approach [4] was specifically developed for plane strain rolling. This approach has been successfully adopted for describing the rolling process of various materials [5-9]. Solutions obtained by the approach [4] are used to control the accuracy of other approximate solutions [10, 11]. It is therefore of interest to extend the approach to other metal forming processes. Also, the original paper [4] has mainly dealt with stress equations. In many cases, however, it is more important to determine an accurate through thickness distribution of the velocity field because it has a great effect on the distribution of material properties. A typical example is the formation of a layer of intensive plastic deformation in the vicinity of frictional interfaces [12]. It is known that theoretical solutions based on several rigid plastic models are singular in the vicinity of maximum friction surfaces [13-15]. In particular, the equivalent strain rate approaches infinity near such interfaces. Such behaviour of the equivalent strain rate allows one to introduce the strain rate intensity factor [13]. This factor is involved in several models for describing the evolution of material structure in a narrow layer near frictional interfaces [16, 17]. Therefore, the theoretical results [13-15] can adequately reflect material behaviour in a narrow layer near interfaces with high friction. However, numerical techniques should directly account for the singularity in solution behaviour. In particular, commercial codes cannot be used to calculate the velocity fields in the vicinity of maximum friction surfaces that follow from the exact equations of rigid plastic models considered in [13-15]. Therefore, the approach [4] enriched with a procedure for finding the velocity field can provide an efficient tool for this kind of problems. The present paper concerns with the distribution of the equivalent strain rate and the strain rate intensity factor in axisymmetric extrusion and drawing of a round rod through a die of arbitrary shape.

2. GENERAL THEORY

Geometry of the process of axisymmetric extrusion/drawing of a round rod is illustrated in Figure 1. It is convenient to introduce the cylindrical coordinate system $r\theta z$ shown in Figure 1. The z-axis coincides with the axis of symmetry. The plane $z = 0$ corresponds to the exit cross-section.
The shape of the die is known and, therefore, the function $h(z)$ is given with $h(0)=\alpha$ and $h(L)=b$. It is assumed that $dh/dz > 0$ in the range $0<z<L$. Here a is the radius of the rod at the exit and b is its radius at the entrance. In the case of extrusion $Q=0$ and in the case of drawing $P=0$. The maximum friction law is supposed on the surface $r=h(z)$. In the case of rigid perfectly/plastic material considered in the present paper this law states that the friction stress at sliding is equal to the shear yield stress k. The method proposed in [4] consists of two main steps. The first step is to solve a boundary value problem for a representative element for determining the distribution of field variables within a generic cross-section. The second step is to use this solution for finding the distribution of the field variables along the plastic zone (along the axis of symmetry in the case under consideration). For plane strain rolling, Prandtl's solution for compression of a block between parallel platens and its generalization have been adopted as stress solutions for the representative element in [4]. An axisymmetric analogue to Prandtl's solution, extrusion from a contracting cylindrical container, has been considered in [18]. This solution can be used in conjunction with Orowan's method to deal with axisymmetric extrusion/drawing processes of

round rods. In particular, it can serve as the solution for the representative element shown in Figure 2. For finding the strain rate intensity factor it is sufficient to consider the velocity solution. The velocity field in the cylindrical coordinates is given by [18]

$$\frac{u_r}{U} = -\frac{r}{h}, \quad \frac{u_z}{U} = \frac{2z}{h} - 2\sqrt{3}\sqrt{1-\frac{r^2}{h^2}} + C \tag{1}$$

where u_r is the radial velocity and u_z is the axial velocity. The solution (1) satisfies the boundary conditions $u_r = 0$ at $r = 0$ (axis of symmetry) and $u_r = -U$ at $r = h$ (maximum friction surface).

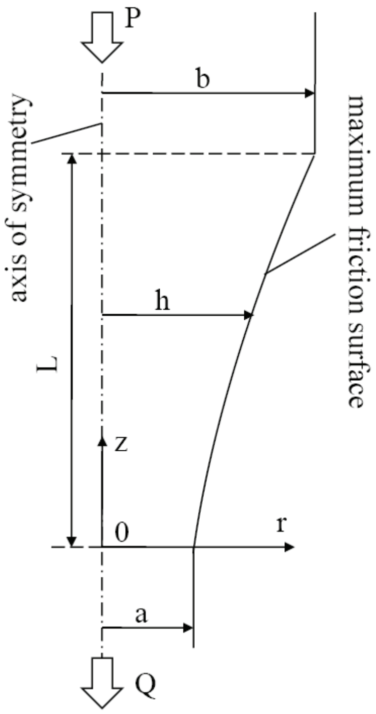

Figure 1 - Geometry of the process

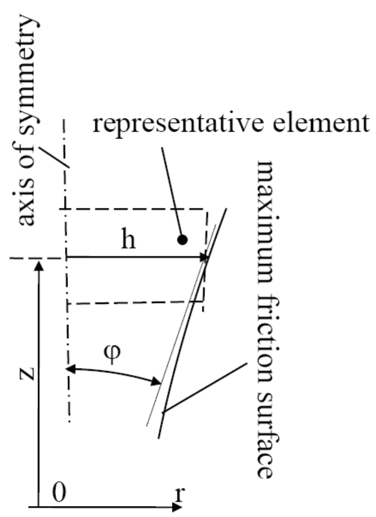

Figure 2 - Representative element

Using the velocity field (1) the components of the strain rate tensor in the cylindrical coordinates can be found in the following form

$$\xi_{rr} = -\frac{U}{h}, \quad \xi_{\theta\theta} = -\frac{U}{h}, \quad \xi_{zz} = \frac{2U}{h}, \quad \xi_{rz} = \frac{\sqrt{3}Ur}{h\sqrt{h^2 - r^2}} \tag{2}$$

In the original solution U and h are prescribed constants and C is a constant of integration. However, in accordance with Orowan's method, it is necessary to assume that all of these quantities are functions of z. Thus (1) predicts the through thickness distribution of the velocity field at a generic cross-section of the plastic zone (Figure 1) and it can be rewritten, with no loss of generality, in the form

$$\frac{u_r}{U} = -\frac{r}{h}, \quad \frac{u_z}{U} = A - 2\sqrt{3}\sqrt{1 - \frac{r^2}{h^2}} \tag{3}$$

where A is a function of z to be found from the solution. It is possible to verify by inspection that (2) near the surface $r = h$ follows the general asymptotic behaviour of the velocity field near maximum friction surfaces [13]. The direction of flow at $r = h$ requires that $A < 0$.

Assume that the volume flux q is given, and consider a generic cross-section within the plastic zone whose radius is $r = h$. Let $z = z_h$ for this cross-section. Then, in the case of incompressible materials

$$q = -\int_0^{2\pi}\int_0^h u_z\big|_{z=z_h} r dr d\theta \tag{4}$$

Substituting (3) into (4) gives

$$q = \pi U h^2 \left(\frac{4}{\sqrt{3}} - A\right) \tag{5}$$

This equation provides one relation between U and A. The other relation can be obtained by considering the direction of the actual velocity vector **u** and the components of the velocity vector from the solution for the representative element, u_r and u_z, at the friction surface $r = h(z)$. It follows from Figure 3 that

$$\cot\varphi = \frac{\left|u_z\big|_{z=z_h}\right|}{U} \tag{6}$$

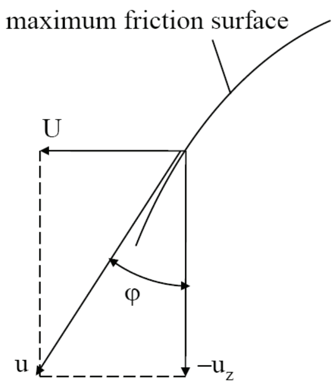

Figure 3 – Components of the velocity vector

Substituting (3) into (6) and taking into account that $A < 0$ gives $\cot\varphi = -A$. Then, it follows from (5) that

$$U = \frac{q}{\pi h^2}\left(\frac{4}{\sqrt{3}} + \cot\varphi\right)^{-1} \tag{7}$$

Since $h(z)$ and $\varphi(z)$ are completely determined by geometry of the die, U as a function of z immediately follows from (7).
A series expansion of the equivalent strain rate in the vicinity of maximum friction surfaces is [13]

$$\xi_{eq} = \frac{D}{\sqrt{s}} + o\left(\frac{1}{\sqrt{s}}\right), \quad s \to 0 \tag{8}$$

where s is the normal distance from the friction surface, D is the strain rate intensity factor, and the equivalent strain rate is expressed in terms of the components of the strain rate tensor ξ_{ij} in the form

$$\xi_{eq} = \sqrt{\frac{2}{3}\xi_{ij}\xi_{ij}} \tag{9}$$

Substituting (2) into (9) gives

$$\xi_{eq} = \frac{\sqrt{6}U}{\sqrt{h^2 - r^2}} \tag{10}$$

In the case under consideration $s = h - r$. Therefore, comparing (8) and (10) and using (7) results in

$$D = \frac{\sqrt{3}q}{\pi h^{5/2}}\left(\frac{4}{\sqrt{3}} + \cot\varphi\right)^{-1} \tag{11}$$

For illustration of the result obtained it is convenient to introduce the dimensionless strain rate intensity factor in the form

$$d = \frac{\pi b^{5/2} D}{q} = \sqrt{3}\left(\frac{b}{h}\right)^{5/2}\left(\frac{4}{\sqrt{3}} + \cot\varphi\right)^{-1} \tag{12}$$

3. EFFECT OF DIE SHAPE ON THE STRAIN RATE INTENSITY FACTOR

Assume that a, b and L are fixed (Figure 1). Then, different process conditions can be obtained by varying the shape of the die. The simplest die is conical. In this case the angle φ is constant (Figure 2) and

$$\tan\varphi = \frac{b-a}{L} \tag{13}$$

Then,

$$h(z) = a + z\tan\varphi \tag{14}$$

Substituting (13) and (14) into (12) gives

$$d = \sqrt{3}\left[\frac{a}{b} + \frac{z}{L}\left(1-\frac{a}{b}\right)\right]^{-5/2}\left(\frac{4}{\sqrt{3}} + \frac{L}{b-a}\right)^{-1} \tag{15}$$

It is assumed now that the shape of the die is given by a polynomial of second order

$$h(z) = a + \alpha\frac{z}{L} + (b-a-\alpha)\left(\frac{z}{L}\right)^2 \tag{16}$$

It is obvious from (16) that $h(0) = a$ and $h(L) = b$. The condition $dh/dz \ge 0$ is satisfied if

$$0 \le \frac{\alpha}{b} \le 2\left(1-\frac{a}{b}\right) \tag{17}$$

It follows from (16) that

$$\tan\varphi = \frac{\alpha}{L} + 2\frac{z}{L^2}(b-a-\alpha) \tag{18}$$

Substituting (16) and (18) into (12) gives

$$d = \sqrt{3}\left[\frac{a}{b} + \frac{\alpha}{b}\frac{z}{L} + \left(1-\frac{a}{b}-\frac{\alpha}{b}\right)\left(\frac{z}{L}\right)^2\right]^{-5/2}\left\{\frac{4}{\sqrt{3}} + \left[\frac{\alpha}{L} + 2\frac{zb}{L^2}\left(1-\frac{a}{b}-\frac{\alpha}{b}\right)\right]^{-1}\right\}^{-1} \tag{19}$$

4. NUMERICAL RESULTS

Let $a/b = 1/2$ and $L/b = 3$. Then, as follows from (17), two extreme cases of die geometry are obtained at $\alpha/b = 0$ and $\alpha/b = 1$. In the case of $\alpha/b = 0$, the tangent to the die is parallel to the z-axis at the exit cross-section. In the case of $\alpha/b = 1$, the tangent to the die is parallel to the z-axis at the entrance cross-section. The variation of the strain rate intensity factor with z/L for these two extreme cases found with the use of (19) as well as for the conical die found by means of (15) is depicted in Figure 4. The curve 1 corresponds to the conical die, curve 2 to the die with $\alpha/b = 0$ and curve 3 to the die with $\alpha/b = 1$. Geometry of the dies is illustrated in Figure 5.

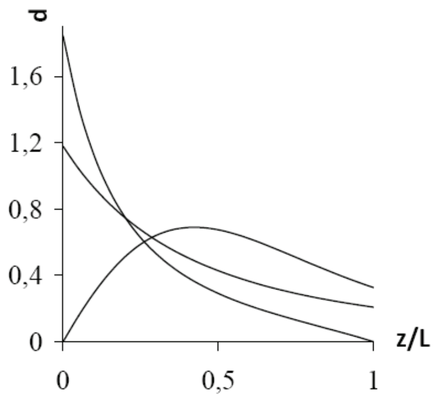

Figure 4 - Variation of the strain rate intensity factor

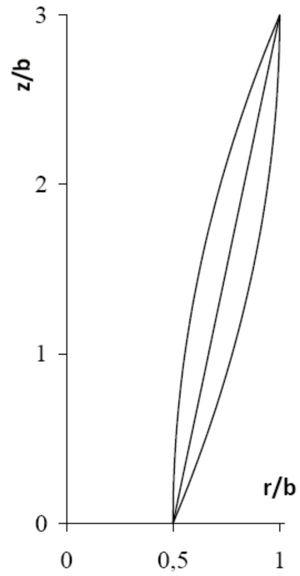

Figure 5 - Geometry of the dies

It is seen from Figure 4 that the strain rate intensity factor significantly varies along the friction surface. This dependence may or may not be monotonic. Geometry of the die strongly influences the magnitude of the strain rate intensity factor giving a room for process design driven by the quality of a narrow sub-surface layer. To this end, it is of course necessary to use a theory that relates the magnitude of the strain rate intensity factor to the evolution of parameters characterizing material properties. The sensitivity of the magnitude of the strain rate intensity factor to die geometry is illustrated in Figure 6. In this figure solid curves 1, 2 and 3 coincide with the curves shown in Figure 4. The dashed curves correspond to several values of α/b of the interval (17).

It is seen from Figure 6 that a small variation of die shape results in a significant change in the magnitude of the strain rate intensity factor. Finally, to illustrate an effect of the billet radius on the magnitude of the strain rate intensity factor at the same radius of the product and the value of L, it is assumed that $L/a = 6$. Varying the value of a/b the magnitude of the strain rate intensity factor has been calculated for the conical die according to (15). The result is shown in Figure 7. It is seen from this figure that the magnitude of the strain rate intensity factor is also very sensitive to the radius of the billet.

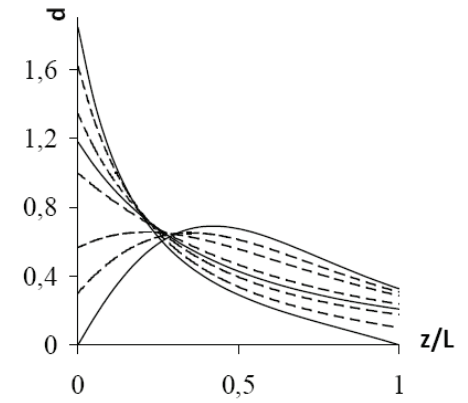

Figure 6 - The sensitivity of the magnitude of the strain rate intensity factor to die geometry

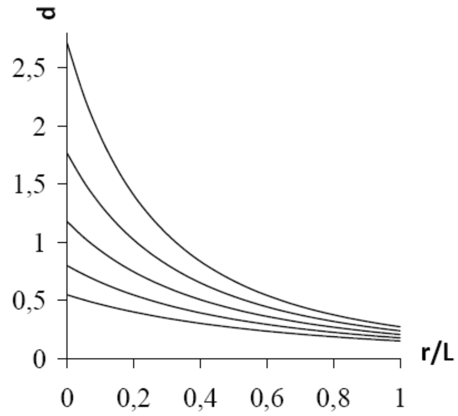

Figure 7 - The sensitivity of the magnitude of the strain rate intensity factor to billet radius

5. CONCLUSIONS

The method proposed in [4] for plane strain rolling has been extended to axisymmetric extrusion and drawing of round rods. The main purpose of the present paper is to use the new method for finding the strain rate intensity factor. Since this factor is associated with the singular velocity field (8), it is important that the method is based on the exact solution for the representative element (Figure 2). For this reason, the particular solution (10) satisfies the general asymptotic representation of the equivalent strain rate (8) and, therefore, the strain rate intensity factor can be evaluated from the approximate solution. Several illustrative examples (Figures 4 to 7) demonstrate that the magnitude of the strain rate intensity factor is sensitive to process parameters, including geometry of the die. It is therefore expected that the method can be used for the process design driven by the quality of a narrow sub-surface material layer. To this end, it is of course necessary to adopt models which relate the magnitude of the strain rate intensity factor and the evolution of parameters characterizing material properties. Such models have been proposed, for instance, in [16, 17]. It is straightforward to further extend the method to treat extrusion of tubes on a mandrel. To this end it is sufficient to replace the velocity field (1) with the general velocity field found in [19] where, however, the appropriate boundary condition (maximum friction law) has to be taken into account.

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GENERALIZACIJA OROVANOVOG PRISTUPA ZA SLUČAJ VALJANJA SA RAVANSKOM DEFORMACIJOM NA PROCESE OSNO SIMETRIČNOG IZVLAČENJA I ISTEZANJA

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REZIME

Približne teorijske metode uspešno se mogu koristiti za preliminarno planiranje procesa oblikovanja metala. Prilikom korišćenja ovih metoda, naročito je važnosti uzeti u obzir ponašanje realnih funkcija koje moraju postojati na mestima gde su lokalni efekti izraženi. U tom smislu, tipičan primer je oblast u kojoj se javlja kontaktno trenja. Poznato je da se stvarno polje brzine neposredno pored površine sa maksimalnim trenje, u slučaju kada se koristi model kruto plastičnog tela, obično definiše preko ne-diferentabilnih funkcija. Osnovna ideja metode predložene od strane Orovana za ravansko deformaciono stanje pri valjanju, upravo je pogodna za opisivanje ovakvih stanja, i u ovom radu ta metoda je primenjena na procese osno-simetričnog izvlačenja i istezanja. Asimptotsko ponašanje stvarnog polja brzine u oblasti u kojoj se javlja trenje je uzeto u obzir kod približne metode. Stoga je moguće proceniti intenzitet faktora brzine deformacije i njegovu zavisnost od parametara procesa. Po pretpostavci, ovaj faktor utiče na osobine materijala u uskom sloju u oblasti površine sa visokim trenjem. Zbog toga se ova metoda može potencijalno primeniti pri projektovanju procesa oblikovanja metala koji zavise od osobina materijala u tankom sloju neposredno ispod površine.

Ključne reči: *Orovanova metoda, polje brzine, osnosimetrično izvlačenje i istezanje*