Original scientific paper
UDK: 655.004.932.2

Evaluation of single and multi-threshold entropy-based algorithms for folded substrate analysis

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Abstract:
This paper presents a detailed evaluation of two variants of Maximum Entropy image segmentation algorithm (single and multi-thresholding) with respect to their performance on segmenting test images showing folded substrates. The segmentation quality was determined by evaluating values of four different measures: misclassification error, modified Hausdorff distance, relative foreground area error and positive-negative false detection ratio. New normalization methods were proposed in order to combine all parameters into a unique algorithm evaluation rating. The segmentation algorithms were tested on images obtained by three different digitalisation methods covering four different surface textures. In addition, the methods were also tested on three images presenting a perfect fold. The obtained results showed that Multi-Maximum Entropy algorithm is better suited for the analysis of images showing folded substrates.

Keywords: Maximum Entropy, image segmentation, folding quality evaluation

Introduction

Image segmentation algorithms are widely used in various fields including optical character recognition, machine vision systems, infrared gait recognition, automatic target recognition and medical image applications (Zhang and Wu, 2011). There are many segmentation methods present, but every algorithm has its strengths and weaknesses. Previous research show their detailed evaluation using objective parameters (Sezgin and Sankur, 2004), (Chang et al., 2006) and (Zhang et al., 2008)**.** One of the main scope of image processing is the field of defect detection. The use of a certain algorithm is based mainly on the type and characteristics of the examined product or process and there is no universal solution. There are many parameters which influence the choice of the used algorithms, for example the expected colours, the complexity of the texture, lighting conditions, the shape or type of failure or its differentiability, etc. (Ng, 2006; Park et al., 2009).

Folding process of the coated papers often causes cracking of the coating layer on the outer side of the folding line (Barbier, 2004). This effect is even more emphasized when fold is made on printed area (surface covered with printing ink). Many factors which influence the crack size in the surface during the folding process are related to the paper manufacturing, to the printing and the converting processes. Even though this phenomenon cannot be completely avoided, the visible

First received: 25.8.2011.
Accepted: 5.10.2011.

destructions of the products can be diminished with more precise folding machine settings (Eklund et al., 2002; Gidlöf et al., 2004; Anon., 2006). Since the online examination of folding quality is still not automated, the derived quality estimation is highly dependent of the examiners experience, fatigue and other subjective influences. The introduction of an automated and objective evaluation based on image processing would probably result in a significant quality improvement and enable process standardization. The objective folding quality assessment algorithm (OFQA) introduced by (Apro et al., 2009) showed promising results. Since most of the measures (model objective variable or MOV) of the proposed algorithm depend on the segmentation quality, detailed evaluation and refinement were performed.

The evaluation, which incorporated 24 different image segmentation algorithms, presented in (Apro et al., 2011), showed that the entropy based algorithms (Maximum and Renyi Entropy algorithms) are the best suited for evaluation of online folding quality assessment. As analysed algorithms are expected to work in a wide range of samples autonomously without any human interaction the most important requirement put upon them was automation of their operation. In order to test the algorithms under the same conditions evaluation of the algorithms was made examining only the single threshold case. However, the selected (Maximum Entropy) algorithm (among others) is capable of multi-thresholding, which could be useful in the folding quality assessment. Namely, the images of folded substrates are potentially three-colour images: the colour of the print, the colour of base paper and the colour of "shadow" (or under-illuminated regions). The aim of this paper is to compare the performance of single (bi-level) and multi (2 thresholds) thresholding approaches using the Maximum Entropy algorithm over a given set of images.

Methods and materials

Thresholding is an important technique for image segmentation that tries to identify and extract the object of interest from its background based on the grey-level distribution or texture in image areas. One of the most efficient techniques for image thresholding is based on entropy distribution of the grey-levels in an image (Liao et al., 2001).

The previous investigation of the authors resulted in selection of Maximum Entropy thresholding as the best performing over the selected set of images. The Maximum Entropy thresholding method (in the form that has been used in this paper), has been proposed by Kapur, Sahoo and Wong and in the literature it is mostly named after its authors (e.g. Liao et al., 2001; Yin, 2002; de Albuquerque et al., 2004; Chang et al., 2006; Zhang et al., 2008; Xiao et al., 2008).

The Maximum Entropy is an automatic thresholding method where the optimal threshold value can be found by maximizing the entropy of the resulting classes (foreground and background) (Chang et al., 2006). This thresholding technique is classified as bi-level approach, where a unique threshold value is obtained. The bi-level segmentation techniques give satisfactory results on the images with clear foreground-background differentiation, but for the segmentation of complex images a multi-thresholding approach could be more suitable. A multi-thresholding technique converts the different types of regions of the image into regions having the optimal number of grey-level (Strouthopoulos and Papamarkos, 2000; Tabbone and Wendling, 2003). The Maximum Entropy approach is one of the most important threshold selection methods but lacks in terms of execution time when the Maximum Entropy criterion is applied to multi-level threshold selection (Luo and Tian, 2000).

The concept of entropy has been widely used in data compression to measure information content of a source, using the uncertainty as a measure to describe the information contained in a source. In image analysis, the entropy-based thresholding considers an image as an information source with a probability vector described by its grey-level image histogram (Chang et al., 2006; Barbieri et al., 2011).

Suppose that h(i) is a value in a normalized histogram. Typically *i* takes integer values from 0 to 255 (for 8-bit depth images). It is assumed that h(i) is normalized, that is (Jarek, 2004):

$$\sum_{i=0}^{i_{\max}} h(i) = 1 \tag{1}$$

The entropy of black pixels is defined as (Jarek, 2004):

$$H_B(t) = -\sum_{i=0}^{t} \frac{h(i)}{\sum_{j=0}^{t} h(j)} \log \frac{h(i)}{\sum_{j=0}^{t} h(j)} \tag{2}$$

The entropy of white pixels is defined as (Jarek, 2004):

$$H_W(t) = -\sum_{i=t+1}^{i_{\max}} \frac{h(i)}{\sum_{j=t+1}^{i_{\max}} h(j)} \log \frac{h(i)}{\sum_{j=t+1}^{i_{\max}} h(j)} \tag{3}$$

The optimal threshold can be selected by maximizing the sum of foreground and background entropies as (Jarek, 2004):

$$T_{opt} = \underset{t=0 \ldots i_{\max}}{ArgMax} \left[H_B(t) + H_W(t) \right] \tag{4}$$

The previous formula for optimal threshold value can be extended to multi-level thresholding of an image. Assuming that there are n thresholds dividing the original image into n+1 classes, the optimal thresholds $\{T_1, T_2, \ldots, T_n\}$ are chosen by maximizing the sum of entropies as follows (Jarek, 2004):

$$\{T_1,\ldots T_n\} = \underset{t_1 < \ldots < t_n}{ArgMax} \left[H(-1,t_1) + H(t_1,t_2) + \ldots + H(t_n, i_{\max}) \right] \tag{5}$$

where:

$$H(t_k, t_{k+1}) = -\sum_{i=t_k+1}^{t_{k+1}} \frac{h(i)}{\sum_{j=t_k+1}^{t_{k+1}} h(j)} \log \frac{h(i)}{\sum_{j=t_k+1}^{t_{k+1}} h(j)} \tag{6}$$

ImageJ software was used for the segmentation process. This software has been chosen as it is widely used, well-supported and free of charge. The bi- and multi-level entropy thresholding algorithms named as “MaxEntropy” (implemented by Jarek S.) and “MultiMaxEntropy” (implemented by K. G. Baler, G. Landini and W. Rasband) can be found at (ImageJ, 2011).

Since, there are three grey-levels expected corresponding to the three-colour surfaces on the test images (the colour of the print, the colour of base paper and the colour of “shadow”), the Multi-Maximum Entropy algorithm was configured to search for two thresholds. As the foreground objects (surface damages) the lightest class was used, the remaining two classes were treated as background. In this way, the results obtained by the two used algorithms (Maximum Entropy and Multi-Maximum Entropy with single and multi-thresholds, respectively) can be compared.

Test images

Test images for the thresholding evaluation process were selected as a subset from a large image library prepared for the needs of the OFQA algorithm development (Apro et al., 2009). A test form was developed based on the experimental test form presented in (Eklund et al., 2002) and (Gidlöf et al., 2004). The test form included test fields for printing quality assurance and for the folding behaviour (surface damage) evaluation. Using the CMYK notation the regions for folding behaviour analysis and visual inspection were as follows: C50% (latter on referred as R1), K50% (R2), K100% (R3), C40% + M40% + Y50% + K20% (R4), C80% + M80% + Y80% + K80% (R5).

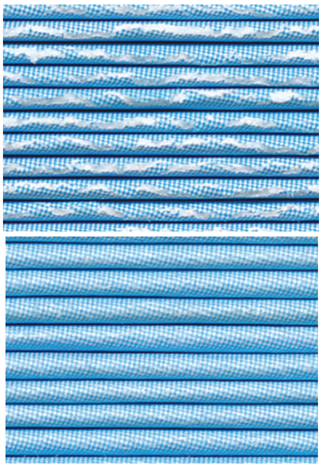
The samples were made of uncoated, glossy- and matte-coated paper (FEDRIGONI) with basic weight of 100 g/m², 140/150 g/m² and 170 g/m², respectively. The printing was performed by KBA Performa 74 offset machine using process colours (Sun Chemical) while the folding was performed by the Horizon AF-C546AKT folding machine. The folding process was made using one buckle folding unit with standard fold rollers (combination of soft polyurethane foam rubber and steel roller) and roller gap adjustment. 50 samples of each paper grade were folded in machine and cross direction of paper grain at temperature of 22 ºC and relative humidity of 55% 48 hours after printing. The prepared (folded and gathered) samples have all been digitalised three times using different digitalisation techniques: commercial digital camera, flatbed scanner and digital microscope. Technical parameters and adjustments for used equipment are presented in Table 1.

Twelve images were chosen from the base set of test images covering all three digitalisation methods and four different surface textures (R1 to R4). Region R5 was excluded from the evaluation process, since it has been used to analyze the folding behaviour at total ink coverage of 320% and its visual appearance was very similar to region R3. All selected test images were of substrates with surface damages. To present the perfect folds without surface damages three more images (one

Table 1. Technical parameters and adjustments for used digitalisation equipment

Used equipment	Canon A520	CanoScan 5600F	Veho VMS-001
Type	Commercial digital camera	Flatbed scanner	USB digital microscope
Colour mode	RGB	RGB	RGB
Embedded colour profile	sRGB	sRGB	-
Resolution	resolution 180 ppi	resolution 1200 ppi	resolution 300 ppi
Bit depth	8	8	8
Format	JPEG	BMP	BMP
Other	no flash, 100% digital zoom, auto white balance focal length of 5,8 mm	-	no light source, magnification of 200X, CMOS sensor

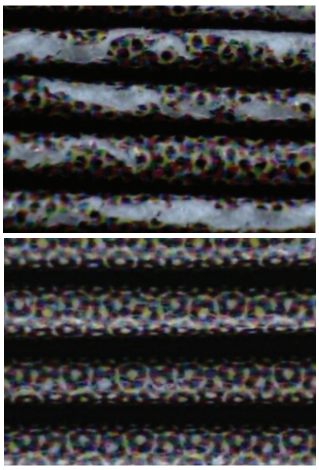
for each digitalisation methods) were added to the evaluation subset in order to proof the over detection rate of the segmentation. The halftone and rosette pattern of printed surfaces, damaged or not, present a particular challenge for automated image segmentation, since the unprinted areas of pattern structures look very similar to cracked surfaces of paper. Samples of halftone and rosette pattern with and without surface damages are shown in Figure 1a-b and Figure 2a-b, respectively.



a)

b)

Figure 1. Test images for halftone pattern of 50% cyan with (a) and without (b) surface damages



a)

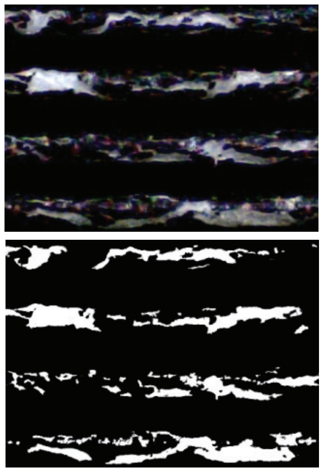
b)

Figure 2. Test images for rosette pattern with (a) and without (b) surface damages

Since, the algorithm works with grey-scale images, the original sample images were transformed from source RGB colour space to CIE xyY colour space which lightness channel (Y) was used as the grey-scale representation. Test images were pre-filtered with Mean filter (using a 3x3 square kernel) in order to reduce the influence of noise and complex patterns (halftone and rosette) on the segmentation results.

Objective measures

To evaluate the segmentation performance of the two variants of Maximum Entropy, four performance measures have been used: misclassification error, modified Hausdorff distance, positive-negative false detection ratio and relative foreground area error. All these measures require a referent or ground truth image, which is derived by hand segmenting every sample image, marking just the cracked surfaces as foreground objects (see Figure 3a and b for an example of test image and its ground truth pair).



a)

b)

Figure 3. Example of test image and its ground truth pair

The proposed measures were chosen in order to optimise the following criterion: automated segmentation results should match the ground truth image as close as possible, preferably 100%, but if there are some mismatches, the misclassified pixels should be as close as possible to the desired foreground object. Since some measures of the OFQA are using the ratio of foreground and background pixels, it is also preferable to have balanced number of false positive and false negative detections, which would lead to good estimation of the real value.

Misclassification error (ME) reflects the percentage of background pixels wrongly assigned to foreground, and vice versa. For the two class segmentation problem, ME can be expressed as (Sezgin and Sankur, 2004):

$$ME = 1 - \frac{|B_O \cap B_T| + |F_O \cap F_T|}{|B_O| + |F_O|}, \tag{7}$$

where B_O and F_O denote the background and foreground of the ground truth image, B_T and F_T denote the background and foreground areas of the tested image, and is the cardinality of the set. ME varies from 0 for a perfectly classified image to 1 for a total mismatch between referent and tested image (Sezgin and Sankur, 2004).

The *Hausdorff distance* can be used to assess the shape similarity of the thresholded regions to the ground truth shapes. Since the maximum distance is sensitive to outliers, shape distortion can be measured via the average of the *Modified Hausdorff distances* (MHD) over all objects, which is defined as (Sezgin and Sankur, 2004):

$$MHD(F_O, F_T) = \frac{1}{|F_O|} \sum_{f_O \in F_O} d(f_O, F_T), \tag{8}$$

where $d(f_O, F_T)$ denotes the minimal Euclidean distance of a pixel in the thresholded image from any pixel in the ground-truth image, $|F_O|$ is the number of foreground pixels in the ground-truth image (Sezgin and Sankur, 2004). Since an upper bound for the Hausdorff distance cannot be established, the MHD metric is hard to normalize to the interval [0, 1]. (Sezgin and Sankur, 2004) divided the derived MHD measures by the maximal value of MHD for the given test image set, but this method gives a relative result, which is sensitive to the choice of the image set. Namely, if the original set is extended with an image, which has a large MHD, this would result in “improvement” in the rest of the pictures for this measure. For this reason a new normalization was proposed in this research. If only the misclassified pixels are used for computing the MHD, its minimum value is 1, while the maximum is not determinable. Converting this value range to values between 0 and 1 can be performed by computing the reciprocate value. However, this would convert the best possible value to 1 (1/1) and the worst value ≈ 0 (1/high value). To keep the same notation (0 for best, 1 for the worst performance), this could be inversed by the following formula:

$$NMHD = 1 - \left(\frac{1}{MHD} \right) \tag{9}$$

However, this formula has a rather steep slope (e.g. for an MHD of 2, which is very close to the best performance, would result in 0.5 which is a considerable distance to 0), thus the following modification was used:

$$NMHD = 1 - \left(\frac{1}{1 + 0.2 \times (MHD - 1)} \right) \tag{10}$$

The measure derived by this formula does not depend on the set of images and has a more general meaning.

The *positive-negative false detection ratio* (PNFDR) is an auxiliary measure to make the evenness of false detection values (positive and negative) easy to read. It is defined as:

$$PNFDR = \begin{cases} \dfrac{PFD}{NFD}, & \text{if PFD} \geq \text{NFD} \\ \dfrac{NFD}{PFD}, & else \end{cases} \tag{11}$$

where: *positive false detection* (PFD) is the proportion of background pixels wrongly assigned to the foreground object. Normalization can be done using the number of foreground pixels or the overall number of pixels in the ground-truth image. In this paper, normalization was done using the number of foreground pixels;

negative false detection (NFD) is the proportion of foreground pixels wrongly assigned to the background. Normalization was performed in the same manner as for the PFD.

The PNFDR measure has a minimum in 1, which is also its optimum, desired value. However, the maximum value cannot be analytically determined, which means that normalization is not strait forward, therefore a similar method is proposed as for the normalization of MHD, that is:

$$NFDR = 1 - \left(\frac{1}{PNFDR} \right) \tag{12}$$

Relative foreground area error (RAE) is first proposed by (Sezgin and Sankur, 2004). It is a comparison of object properties, more specifically the area of the detected and expected foreground. It is defined by the following equation:

$$RAE = \begin{cases} \dfrac{A_0 - A_T}{A_0}, & if\, A_T < A_0 \\ \dfrac{A_T - A_0}{A_T}, & else \end{cases} \tag{13}$$

where A_0 is the area of reference image and A_T is the area of the thresholded image. For a perfect match RAE is 0, while if there is zero overlap of the object areas the RAE is 1.

To obtain a single, joint performance score from the previously listed criteria the arithmetic averaging of four measures has been considered. The normalized scores obtained from the ME, NMHD, NFDR, and RAE criteria were used, while the PFD and the NFD were omitted since their information is contained in their ratio measure (NFDR). The attempt to derive a combined measure was presented by (Sezgin and Sankur, 2004) with the clear attempt to simplify segmentation quality evaluation. However, since the measures are not fully independent, share some portion of the information and their sensitivity (slope of the function) is different, simple arithmetic averaging might not give the best approximate of the overall quality measure. Therefore, the evaluation of tested algorithms was made by using only the ME as a representative measure and by using the joint measure. The two evaluation results were then compared.

Results and discussion

The evaluation of the results was done separately for the ME measure and the combined measure. The ME measure is the most descriptive, it shows the percentage of misclassified pixels, hence is a quantitative description of the error, while the other measures are more qualitative. The combined measure recapitulates the presented four measures into a single one.

The results are presented and analyzed in two different groups: first group includes images with surface damage, second group, the images showing a perfect fold. This separation is made because in the second group most of the measures were not defined, since there is no foreground object on the ground truth images. For this group, only results of the ME measure are shown.

The obtained results of ME calculated on test samples showing damaged surface are presented in Figure 4. According to the ME, as a standalone measure, it can be seen that Multi-Maximum Entropy has a balanced performance over the whole set of test images with maximum ME value of 0.2. On the other hand, the Maximum Entropy had lower ME values for 9 images. The other 3 images had significantly higher ME values than with Multi-Maximum Entropy.

Comparing the performance of segmentation techniques for different digitalisation methods one could see large differences between values obtained using Maximum Entropy to the values obtained using Multi-Maximum Entropy at images of region R4 taken with commercial camera and scanned. Images taken by digital microscope, were well segmented with both algorithms resulting with similar values of ME.

In addition, images of cyan printed substrates (1_mic, 4_scan and 4_cam) were poorly segmented using only one threshold. This could be explained by the relatively small difference in luminance levels of cyan and paper colours. By using two thresholds, these fine differences could be more distinguished. It could be noticed that Multi-Maximum Entropy is more successful in segmenting images where the luminance values of printed substrate colour and paper colour are harder

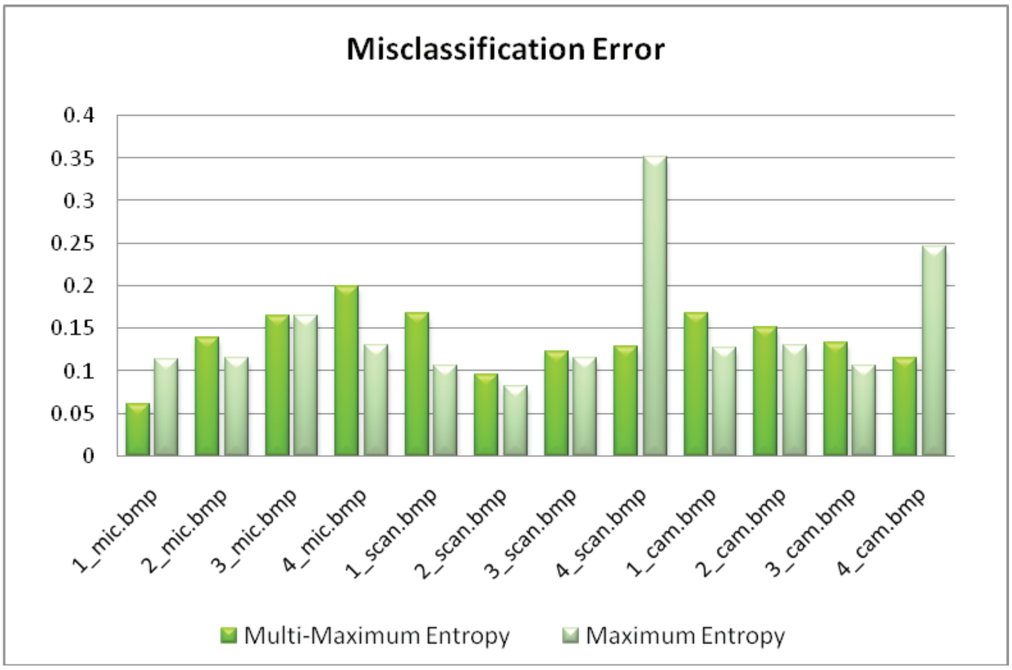


Figure 4. Values of ME for every test images with surface damages

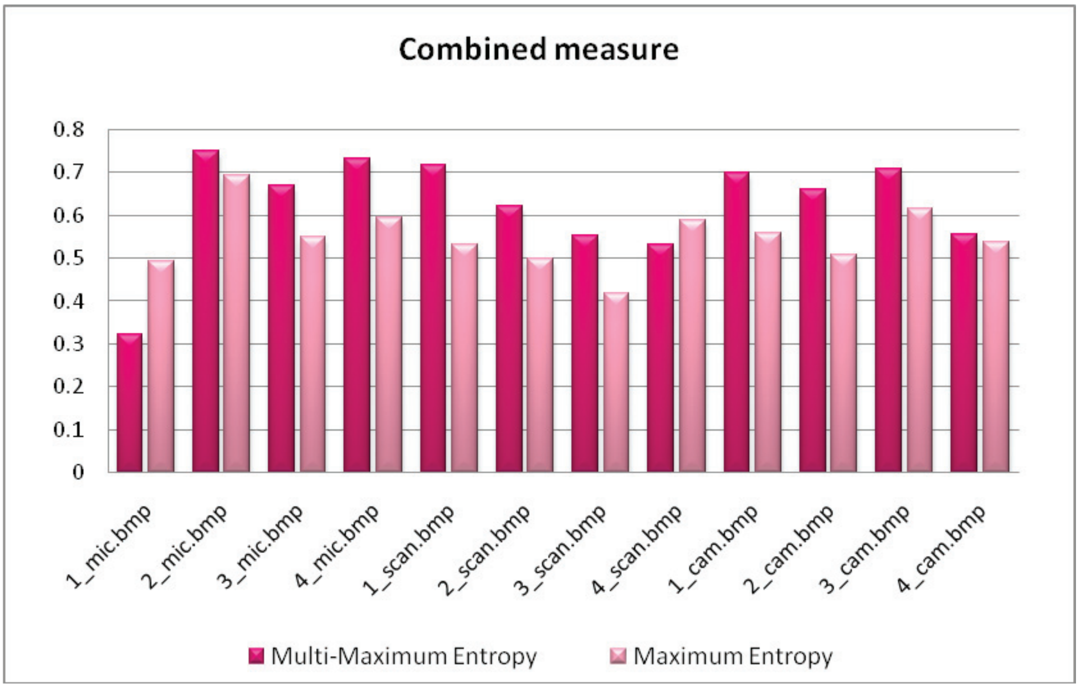


Figure 5. Values of combined measure for every test images with surface damages

to distinguish (1_mic, 4_scan and 4_cam). However, it should also be mentioned that Multi-Maximum Entropy is more prone to under-detecting (false negative detections) the damages, but there is almost no positive false detection. This feature of the algorithm could be well exploited in a two-staged segmentation approach (where the Multi-Maximum Entropy would be used as the first step to mark damage pixels).

In Figure 4 can also be seen that all the digitalisation methods are showing similar results, which lead to the conclusion that substrate inspection can be conducted online in production where optimal illumination is hard to achieve.

Based on the combined measure, the Maximum Entropy showed better performance than Multi-Maximum

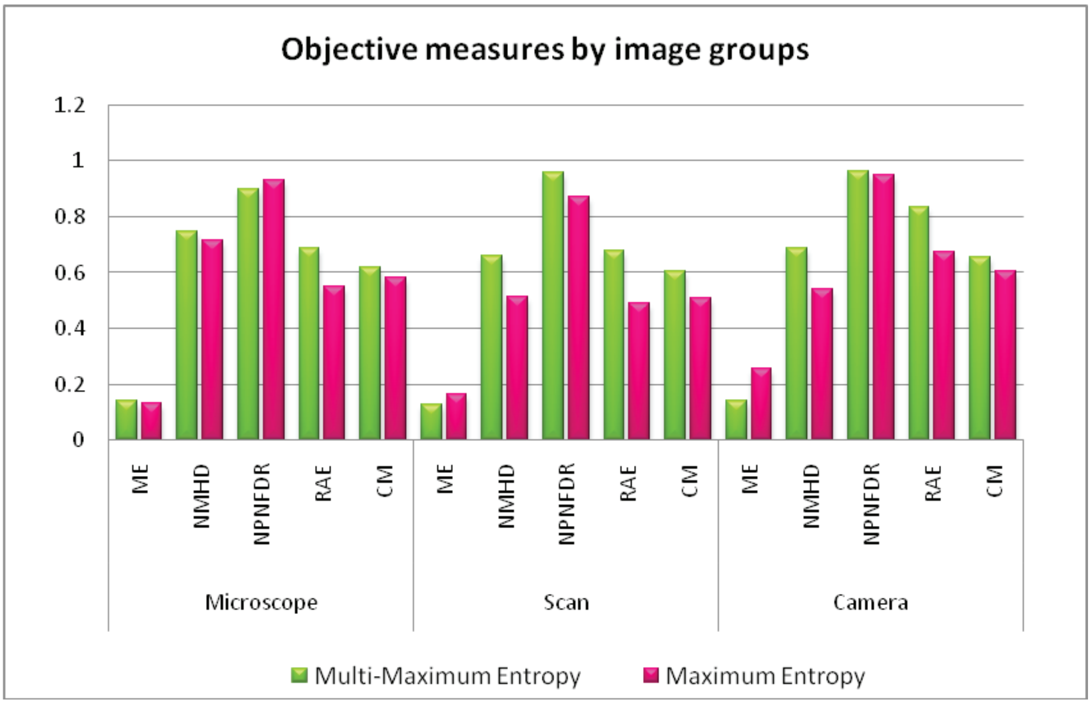


Figure 6. The average values of all objective measures by image groups

Table 2. Values of ME for test images without surface damages

Test image	Multi-Maximum Entropy	Maximum Entropy
5_mic.bmp	0.002573	0.019143
5_scan.bmp	0.001827	0.415236
5_pict.bmp	0.000155	0.000211

Entropy for most of the test images (Figure 5), hence the Maximum Entropy can be considered as a better choice. However, this is opposite to the suitability evaluation of investigated algorithms based on ME values. Although, similar combined measure was successfully used in (Sezgin and Sankur, 2004) results of this investigation indicate that current form is not suitable for usage in algorithm evaluation. Nevertheless, the quantitative measures are also carrying useful information, which should also be utilized, but using a more sophisticated equation. Figure 6 presents an overview of all measures averaged by image groups based on the digitalisation method.

The results for the second group of pictures (substrates without surface damages) are shown in Table 2.

The Multi-Maximum Entropy had better segmentation results for all three investigated images of second group, showing significant improvement on the scanned image. These results are even more important if we consider that these are images of perfect folds, where over-detection is critical. This makes the Multi-Maximum Entropy better algorithm to use for detecting damages on folded substrates. However one should have in mind that the Multi-Maximum Entropy has high computational demands, although this becomes critical when using for three or more thresholds (Luo and Tian, 2000), hence it does not apply to the case considered here as there are only two thresholds used.

Conclusions

This paper presents a detailed evaluation of two variants of Maximum Entropy image segmentation algorithm with the focus to their performance on segmenting a specific group of images. These images are showing different folded test specimens, which are inputs to an objective fold quality assessment algorithm. The algorithm uses image segmentation to detect surface damages and hence the segmentation quality metrics are chosen accordingly. There were four different measures used for algorithm evaluation: misclassification error, modified Hausdorff distance, relative foreground area error, positive-negative false detection ratio. Attempting to combine the measures into a unique rating for algorithm evaluation, new normalization methods were proposed for some of the parameters. Two evaluations of the algorithms were performed, first based only on the ME measure and second based on the combined measure. The first evaluation showed better results for the multi-threshold Maximum Entropy algorithm, while the second gave a different result showing better performance of single threshold Maximum Entropy. In order to resolve this confusion, detailed analysis of partial results was performed, which showed that Multi-Maximum Entropy algorithm is better suited for the folding analysis. Furthermore, the combined measure should be differently constructed in order to give unique and precise rating about algorithm performance. However, this is beyond the scope of this research and will be considered in further research.

Acknowledgments

This work was supported by the Serbian Ministry of Science and Technological Development, Grant No.:35027 “The development of software model for improvement of knowledge and production in graphic arts industry”

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